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Magnetized particle motion around magnetized Schwarzschild-MOG black hole revisited

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Abstract. In this study, we revisit the dynamics of magnetized particle motion in the Schwarzschild-MOG (Modified Gravity) black hole spacetime under the influence of an asymptotically uniform magnetic field. We identify significant discrepancies with prior results, particularly those in [1], where the impact of an external vector field on particle motion around the black hole was overlooked. Our analysis addresses these gaps by exploring the interaction between the magnetic field and massive particles within the framework of Scalar-Tensor-Vector Gravity (STVG). Specifically, we derive a generalized dynamic Lagrangian that includes both the magnetic interaction potential and external vector field effects. Using this formalism, we study the particle dynamics, calculate the Hamilton-Jacobi equation, and analyze the effective potential of massive particles near the black hole. We also extend the BSW (Banados-Silk-West) process to compute the center-of-mass energy (CME) of magnetized particle collisions near the black hole, finding conditions under which the CME may diverge due to overlooked vector field contributions in previous studies. This work highlights the importance of accounting for both magnetic and external vector fields when analyzing particle interactions near Schwarzschild-MOG black holes, providing a more accurate description of magnetized particle motion in modified gravity scenarios.

Key words: Scalar-Tensor-Vector gravity, massive particle motion, Schwarzschild-MOG, centre-of-mass energy (CME).

I. INTRODUCTION

We have revisited the magnetized particle motion in spacetime of the Schwarzschild-MOG black hole in the presence of the asymptotically uniform magnetic field. We have found that there are huge discrepancies with that is obtained so far, in particular, in paper [1]. In this note, we explicitly discuss the motion of a magnetized particle orbiting around a black hole in MOG and determine the collision of pair of magnetized particles including the external vector potential. In Ref. [1], magnetized particle motion around the Schwarzschild-MOG black hole in Scalar-Tensor-Vector Gravity (STVG) has been studied. However, author of the paper has been missed external field which interacts with massive particle and potentially affect to massive particle motion around the black hole in this theory [2–5]. Collosiion of the particle has been discussed in [6]. STVG is a modified theory of gravity that extends general relativity and extensively explored in [2] and the Schwarzschild-MOG spacetime within this theory can be described as [3]: $ds^2 = -fdt^2 + f^{-1}dr^2 + r^2d\Omega$, associated with the vector field: $\phi_t = -\sqrt{\alpha}M/r$, where $f = 1 - 2(1+\alpha)M/r + \alpha(1+\alpha)M^2/r^2$. Here, M is the mass parameter of the black hole and α is MOG parameter in STVG. The outer horizon of the Schwarzschild-MOG black hole is located at $r_+ = M(1+\alpha+\sqrt{1+\alpha}) \geq 2M$.

In recent studies, we are aimed to discuss exploring the potential impact of the vector field magnetized particles motion near black holes within the context of MOG (Modified Gravity). Notice that, the effect of the vector potential on gravitational radiation emitted by these massive particles [7, 8] and on motion of the S2 star orbiting a supermassive black hole [9, 10] has been studied.

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II. THE MODEL, EQUATIONS AND SUMMARY

One of the main difference of the Schwarzschild-MOG spacetime from other static black hole solution is that massive particles deviate from geodesic paths due to the presence of the external vector field in this spacetime. Therefore the Lagrangian for massive particle in STVG is given as [7]

$$L = \frac{1}{2} m g_{\mu\nu} u^\mu u^\nu + q\phi_\mu u^\mu \,, \tag{1}$$

where u^μ is the four-velocity of a test particle with mass m normalized as $u_\mu u^\mu = -1$ and $q = \sqrt{\alpha} m$ is coupling constant of STVG theory, which means $\alpha \geq 0$. On the other hand the dynamic Lagrangian for magnetized particle in the framework of general relativity (GR) is formulated as [11]

$$L = \frac{1}{2}(m + \mathcal{U}) g_{\mu\nu} u^\mu u^\nu - \frac{1}{2} k\mathcal{U} \,, \tag{2}$$

where $\mathcal{U}$ is the potential of magnetic interaction defined as $\mathcal{U} = (1/2) F_{\alpha\beta} P^{\alpha\beta}$ and k is kind of constant. The explicit form of the electromagnetic field tensor $F_{\alpha\beta}$ and polarization tensor $P_{\alpha\beta}$ can be found in [11]. Now our aim is to write generalized dynamic Lagrangian for magnetized particle in STVG. Without losing generality, one can combine Lagrangian in equations (1) and (2) and obtain

$$L = \frac{1}{2}(m + \mathcal{U}) g_{\mu\nu} u^\mu u^\nu + q\phi_\mu u^\mu - \frac{1}{2} k\mathcal{U} \,, \tag{3}$$

and using the definition of the four-momentum of particle $p_\alpha = \partial L / \partial u^\alpha$, the super-Hamiltonian for magnetized particle in STVG then can be easily written as

$$H = \frac{1}{2}\left[\frac{1}{m + \mathcal{U}} g^{\mu\nu} (p_\mu - q\phi_\mu)(p_\nu - q\phi_\nu) + \frac{1}{2} k\mathcal{U} \right] \,. \tag{4}$$

Finally, using the canonical formalism, one can the Hamilton-Jacobi equation (for particular choice of $k = 1$) in a form:

$$g^{\mu\nu} \left(\frac{\partial S}{\partial x^\mu} - q\phi_\mu \right) \left(\frac{\partial S}{\partial x^\nu} - q\phi_\nu \right) = -(m + \mathcal{U})^2 \,. \tag{5}$$

However, in the paper [1] potential terms are missed. It is also important to test the effect of MOG in the magnetic interaction term $\mathcal{U}$, in particular, electromagnetic field tensor $F_{\mu\nu}$ and polarization tensor $P_{\mu\nu}$. Using the Wald approach [12], an exact analytical expression for the vector potential in the Schwarzschild-MOG space has been found as $A_\phi = (1/2) B_0 r^2 \sin^2\theta$ in [1], however proper solution to vector potential has been found as [13]

$$A_\phi = \frac{1}{2} B_0 \left[r^2 - M^2 \alpha (1 + \alpha) \right] \sin^2\theta \,, \tag{6}$$

where B_0 is asymptotically uniform magnetic field.

Utilizing the formalism previously discussed, equation of motion for magnetized particle in the Schwarzschild-MOG spacetime can be derived as (See e.g.[14])

$$\dot{r}^2 = \left(\mathcal{E} - \frac{\alpha M}{r} \right)^2 - f \left(1 + \frac{l^2}{r^2} - \frac{\beta f}{\sqrt{f - \Omega^2 r^2}} \right) \,, \tag{7}$$

$$\dot{t} = \frac{1}{f} \left(\mathcal{E} - \frac{\alpha M}{r} \right) \,, \qquad \dot{\phi} = \frac{l}{r^2} \,, \tag{8}$$

where $\mathcal{E}$, l and $\Omega = d\phi/dt$ are, respectively, specific energy, specific angular momentum and angular velocity of magnetized particle measured by a distant observer, $\beta = 2\mu B_0 / m$ is magnetic coupling constant with magnetic dipole moment of particle μ.

One of main results of the paper [1] is dedicated to determining of centre-of-mass energy (CME) of colliding pair of particles, in particular, magnetized-magnetized, magnetized-charged and charged-charged particles in the vicinity of the Schwarzschild-MOG black hole, based on the mechanism so called BSW process [15]. Here we will focus on the collision of magnetized particles near the Schwarzschild-MOG black hole and CME can be determined as

$$\frac{E^2_{\rm c.m.}}{2(m_1+\mathcal{U}_1)(m_2+\mathcal{U}_2)} = 1 - \frac{(m_2-m_1+\mathcal{U}_2-\mathcal{U}_1)^2}{2(m_1+\mathcal{U}_1)(m_2+\mathcal{U}_2)} - \frac{l_l l_2}{r^2} + \frac{1}{f}\Bigg[\left(\mathcal{E}_1-\frac{\alpha M}{r}\right)\left(\mathcal{E}_2-\frac{\alpha M}{r}\right) - \sqrt{\left(\mathcal{E}_1-\frac{\alpha M}{r}\right)^2 - f\left(1+\frac{l_1^2}{r^2}-\frac{\beta_1 f}{\sqrt{f-\Omega^2 r^2}}\right)} \times\sqrt{\left(\mathcal{E}_1-\frac{\alpha M}{r}\right)^2 - f\left(1+\frac{l_1^2}{r^2}-\frac{\beta_2 f}{\sqrt{f-\Omega^2 r^2}}\right)}\Bigg], \tag{9}$$

On can find that CME of colliding pair of identical magnetized particles near horizon of the Schwarzschild-MOG black hole reads

$$\frac{E^2_{\rm c.m.}}{(m+\mathcal{U})^2} = 2 - \frac{2l_l l_2}{r_+^2} + \frac{\mathcal{E}_1 r_+ - \alpha M}{\mathcal{E}_2 r_+ - \alpha M}\left(1+\frac{l_2^2}{r_+^2}\right) + \frac{\mathcal{E}_2 r_+ - \alpha M}{\mathcal{E}_1 r_+ - \alpha M}\left(1+\frac{l_1^2}{r_+^2}\right), \tag{10}$$

which will be infinity when specific energy of one of particles should be $\mathcal{E} = \alpha M/r_+$, while for particles with identical specific energy $\mathcal{E}_1 = \mathcal{E}_2 = 1$, CME is determined as

$$E^2_{\rm c.m.} = \left[4 + \frac{(l_2-l_1)^2}{r_+^2}\right](m+\mathcal{U})^2, \tag{11}$$

which is finite. However, a critical finding in Ref. [1] demonstrated that, even in this case, the computation leads to the production of an infinite CME for magnetized particles. The primary issue identified was the omission of the vector field ϕ_μ and f^{-1} factor in front of the square root term in expression (9). Similar oversights were also made in the study of collisions involving charged-charged and charged-magnetized particles near the Schwarzschild-MOG black hole.

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